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# Effect of non-uniform slot injection (suction) on a forced flow over a slender cylinder

Prabal Datta [a], D. Anilkumar [b], S. Roy [b,*], N.C. Mahanti [a]

[a] *Department of Applied Mathematics, B.I.T Mesra, Ranchi 835215, India*
[b] *Department of Mathematics, Indian Institute of Technology Madras, Chennai 600 036, India*

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**Abstract**

An analysis is performed to obtain the non-similar solution of a steady laminar forced convection boundary layer flow over a horizontal slender cylinder including the effect of non-uniform slot injection (suction). The effects of transverse curvature and viscous dissipation are also included in the analysis. The governing boundary layer equations along with the boundary conditions are first cast into a dimensionless form using suitable transformations and the resulting system of nonlinear coupled partial differential equations is then solved by an implicit finite difference scheme in combination with the quasilinearization technique. Numerical results for the effect of non-uniform slot injection (suction) on skin friction coefficient and heat transfer rate are presented. The effects of transverse curvature, viscous dissipation and Prandtl number on velocity and temperature profiles and skin friction and heat transfer coefficients are also reported.


## 1. Introduction

Flows over cylinder are usually consider to be two dimensional as long as the body radius is large compared to the boundary layer thickness. On the other hand for slender cylinder, the radius of the cylinder may be of the same order as that of the boundary layer thickness. Therefore, the flow may be consider as axisymmetric instead of two dimensional. In such a case, the governing equations contain the transverse curvature term which strongly influences the velocity and temperature fields and correspondingly the skin friction coefficient and heat transfer rate at the wall. Among the earlier studies, the magnitude of the transverse curvature effect has been investigated for isothermal laminar flows by Cebeci [1] and the results show that the local skin friction can be altered by an order of magnitude due to an appropriate change in the ratio of boundary layer thickness to cylinder radius. Further, Chen and Mucoglu [2] and Mucoglu and Chen [3] have investigated buoyancy effects on forced convection flow along vertical cylinder for uniform wall temperature and uniform heat flux conditions, respectively. Subsequently Bui and Cebeci [4], Wang and Kleinstreuer [5], and recently, Takhar et al. [6] have studied the combined effect of free and forced convection flows over vertical slender cylinder. It is therefore evident that the calculations of momentum and heat transfer on slender cylinders should consider the transverse curvature effect, especially in applications such as wire and fiber drawing, where accurate predictions are required and thick boundary layers can exist on slender or near-slender bodies.

In many cases of interest, mass transfer from a wall slot (i.e., mass transfer occurs in a small porus section of the body surface while there is no mass transfer in the remaining part of the body surface) into the boundary layer is of interest for various potential applications including thermal protection, energizing of the inner portion of boundary layer in adverse pressure gradient, and skin friction reduction on control

* Corresponding author. Fax: +91 044 2257 4602.
*E-mail address:* sjroy@iitm.ac.in (S. Roy).

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**Nomenclature**

| | |
|---|---|
| $A$ | surface mass transfer parameter |
| $C_f$ | local skin friction coefficient |
| $C_p$ | specific heat at constant pressure |
| $Ec$ | Eckert number |
| $f$, $F$ | dimensionless stream function, velocity component |
| $G$ | dimensionless temperature |
| $k$ | thermal conductivity |
| $Nu$ | local Nusselt number |
| $Pr$ | Prandtl number |
| $r$ | radial coordinate |
| $Re_x$ | Reynolds number |
| $r_0$ | radius of cylinder |
| $T$ | temperature |
| $u$ | axial velocity component |
| $U$ | free stream velocity component |
| $v$ | radial velocity component |
| $x$ | axial coordinate |
| $w^*$ | slot length parameter |

*Greek symbols*

| | |
|---|---|
| $\eta$ | similarity variable |
| $\mu$ | dynamic viscosity |
| $\nu$ | kinematic viscosity |
| $\xi$ | transverse curvature |
| $\rho$ | density |

*Subscripts*

| | |
|---|---|
| w, $\infty$ | conditions at the wall and infinity, respectively |
| $\xi$, $\eta$ | denote the partial derivatives w.r.t. these variables, respectively |

surfaces. In fact, mass transfer through a slot strongly influences the development of a boundary layer along a surface and different studies [7–9] show the effect of slot injection (suction) into a laminar compressible boundary layer over a flat plate by considering the interaction between the boundary layer and oncoming stream. Uniform mass transfer in a slot causes finite discontinuity at the leading and the trailing edges of the slot. The discontinuities can be avoided by choosing a non-uniform mass transfer distribution along a streamwise slot as has been discussed in Minkowycz et al. [10] and also in recent investigations by Roy [11] and Roy and Saikrishnan [12,13].

In the present analysis, the influence of non-uniform slot injection (suction) on a flow over a horizontal slender cylinder including the effects of transverse curvature, viscous dissipation and Prandtl number are considered. The present analysis may be useful in understanding many boundary layer flow problems of practical importance because the use of a slender body reduces the drag and even produces sufficient lift to support the body in certain situations. Further several transport processes with surface mass transfer i.e., injection (or suction) in industry where thermal diffusion caused by the temperature gradient such as polymer fiber coating or the coating of wires, etc. may have useful applications of the present study. In this investigation, the non-similar solutions have been obtained starting from the origin of streamwise coordinate using the quasi-linearization technique with an implicit finite difference scheme. There are two free parameters in this problem, one measures the length of the slot (i.e., the part of the body surface in which there is a mass transfer) and another parameter fixes the position of the slot. Thus, these two parameters help to vary the slot length and to move the slot location. It may be also noted that the finite discontinuities at the leading and trailing edges of the slot have been avoided following [10–13]. Thus, the present analysis differs from those in [7–9] with finite discontinuities.

## 2. Analysis

We consider the steady laminar forced convection flow over a horizontal slender cylinder of radius $r_0$ with non-uniform slot injection/suction. The flow is taken to be axisymmetric and Fig. 1 shows the coordinate system and the physical model. The blowing rate is assumed to be small and it does not affect the inviscid flow at the edge of the boundary layer. The effects of transverse curvature and viscous dissipation are also included in the analysis. The fluid at the edge of the boundary layer is maintained at a constant temperature $T_\infty$ and the body has a uniform temperature $T_w$ ($T_w >$ or $< T_\infty$, i.e., the cylinder is either heated or

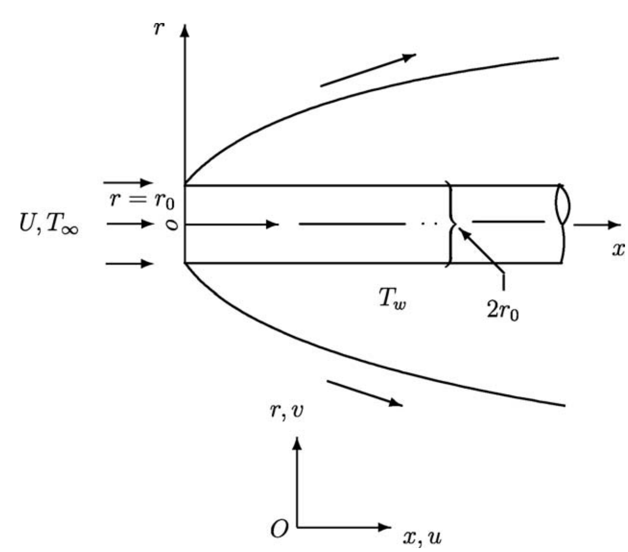


Fig. 1. Physical model and coordinate system.

cooled). It is assumed that the injected fluid posses the same physical properties as the boundary layer fluid and has a static temperature equal to the wall temperature. Under the above assumptions, the governing boundary layer equations can be expressed as [6,14,15]:

$$\frac{\partial(ru)}{\partial x}+\frac{\partial(rv)}{\partial r}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial r}=(v/r)\frac{\partial}{\partial r}\left(r\frac{\partial u}{\partial r}\right) \tag{2}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial r}=\frac{v}{Pr}\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)+\frac{\mu}{\rho C_p}\left(\frac{\partial u}{\partial r}\right)^2 \tag{3}$$

The boundary conditions are given by

$$\begin{aligned}&u(x,r_0)=0,\quad v(x,r_0)=v_{\mathrm{w}}(x),\quad T(x,r_0)=T_{\mathrm{w}}=\text{constant}\\&u(x,\infty)=U,\quad T(x,\infty)=T_\infty=\text{constant}\end{aligned} \tag{4}$$

Applying the following transformations:

$$\begin{aligned}&\xi=\left(\frac{4}{r_0}\right)\left(\frac{vx}{U}\right)^{\frac{1}{2}},\quad \eta=\left(\frac{U}{vx}\right)^{\frac{1}{2}}\left[\frac{r^2-r_0^2}{4r_0}\right],\\&\frac{r^2}{r_0^2}=[1+\xi\eta],\quad \psi(x,r)=r_0(vUx)^{\frac{1}{2}}f(\xi,\eta),\\&u=\frac{1}{2}Uf_\eta=\frac{U}{2}F,\quad v=\frac{1}{2r}r_0\left(\frac{vU}{x}\right)^{\frac{1}{2}}(\eta f_\eta-f-\xi f_\xi),\\&f_\eta(\xi,\eta)=F(\xi,\eta),\quad G(\xi,\eta)=\frac{T-T_\infty}{T_{\mathrm{w}}-T_\infty},\\&Ec=\frac{U^2}{4C_p(T_{\mathrm{w}}-T_\infty)},\quad Pr=\frac{\mu C_p}{k}\end{aligned} \tag{5}$$

to Eqs. (1)–(3), we find that Eq. (1) is satisfied identically, and Eqs. (2) and (3) reduce to

$$(1+\xi\eta)F_{\eta\eta}+(\xi+f)F_\eta=\xi(FF_\xi-F_\eta f_\xi) \tag{6}$$

$$\begin{aligned}&Pr^{-1}(1+\xi\eta)G_{\eta\eta}+(\xi Pr^{-1}+f)G_\eta+Ec(1+\xi\eta)F_\eta^2\\&\quad=\xi(FG_\xi-G_\eta f_\xi)\end{aligned} \tag{7}$$

Here, $\xi$, $\eta$ are the transformed coordinates; $\eta_\infty$ is the edge of the boundary layer; $\psi$ and $f$ are dimensional and dimensionless stream functions, respectively; $F$ and $G$ are, respectively, dimensionless velocity and temperature; $Ec$ is the viscous dissipation parameter or Eckert number; Subscripts $\infty$ and w denote the conditions at the free stream and at the wall, respectively.

The boundary conditions reduce to

$$\begin{aligned}&F(\xi,0)=0,\quad G(\xi,0)=1\quad \text{at } \eta=0\\&F(\xi,\infty)=2,\quad G(\xi,\infty)=0\quad \text{as } \eta\to\infty\end{aligned} \tag{8}$$

where $f=\int_0^\eta F\,\mathrm{d}\eta+f_{\mathrm{w}}$ and $f_{\mathrm{w}}$ is given by

$$f_{\mathrm{w}}=-\left(\frac{r_0}{2v\xi}\right)\int_0^\xi \xi v_{\mathrm{w}}(\xi)\mathrm{d}\xi;$$

$$v_{\mathrm{w}}(\xi)=-\left(\frac{vU}{x}\right)^{\frac{1}{2}}\frac{1}{2}[f+\xi f_\xi]=-\frac{2v}{r_0\xi}[f+\xi f_\xi]$$

Here, the boundary condition $v_{\mathrm{w}}(x)$ is considered in terms of transformed coordinate $\xi$ and $v_{\mathrm{w}}(\xi)$ is taken as sinusoidal function given by

$$\begin{aligned}v_{\mathrm{w}}(\xi)&=-A\left(\frac{2v}{r_0}\right)\omega^*\sin[\omega^*(\xi-\xi_0)],\quad \xi_0\leqslant\xi\leqslant\xi_0^*\\&=0,\quad \xi\leqslant\xi_0\ \text{or}\ \xi\geqslant\xi_0^*\end{aligned}$$

and $f_{\mathrm{w}}$ can be written as

$$\begin{aligned}f_{\mathrm{w}}&=0\quad \text{for } \xi\leqslant\xi_0\\&=\left(\frac{A}{\xi}\right)\phi(\xi_0,\xi)\quad \text{for } \xi_0\leqslant\xi\leqslant\xi_0^*\\&=\left(\frac{A}{\xi}\right)\phi(\xi_0,\xi_0^*)\quad \text{for } \xi\geqslant\xi_0^*\end{aligned}$$

where

$$\phi(\xi_0,\xi)=\xi_0-\xi\cos\{\omega^*(\xi-\xi_0)\}+\frac{1}{\omega^*}\sin\{\omega^*(\xi-\xi_0)\}$$

Here, $\omega^*$ and $\xi_0$ are the two free parameters which determine the slot length and slot location. The function $v_{\mathrm{w}}(\xi)$ is continuous for all values of $\xi$ and it has a non-zero value only in the interval $[\xi_0,\xi_0^*]$. The reason for taking such a type of function is that it allows the mass transfer to change slowly in the neighbourhood of the leading and trailing edges of the slot. The surface mass transfer parameter $A>0$ or $A<0$ according to whether there is a suction or injection. The consequence of avoiding finite discontinuities at the leading and trailing edges of the slot helps to obtain smooth solutions for a large values of the mass transfer parameter $A$ without the difficulties of numerical instability. It may be noted that such difficulties were pointed out in [7,8] for uniform mass transfer studies with finite discontinuities at the leading and trailing edge of the slot. Even though the paper in [9] describes non-uniform slot injection, the mass transfer distribution has a sharp change at the leading and trailing edges of the slot, which has been avoided in the present study for the advantage of computing skin friction and heat transfer coefficients in the case of comparatively higher mass transfer parameter.

The quantities of physical interest are as follows [6,14,15]:

The local skin friction coefficient is given by

$$C_{\mathrm{f}}=\frac{2[\mu\frac{\partial u}{\partial r}]_{\mathrm{w}}}{\rho U^2}=2^{-1}(Re_x)^{-\frac{1}{2}}(F_\eta)_{\mathrm{w}}$$

Thus,

$$Re_x^{\frac{1}{2}}C_{\mathrm{f}}=2^{-1}(F_\eta)_{\mathrm{w}} \tag{9}$$

The local heat transfer rate at the wall in terms of Nusselt number can be expressed as

$$Re_x^{-\frac{1}{2}}Nu=-2^{-1}(G_\eta)_{\mathrm{w}} \tag{10}$$

where $Nu=-\frac{\left[x\left(\frac{\partial T}{\partial r}\right)\right]_{\mathrm{w}}}{T_{\mathrm{w}}-T_\infty}$.

## 3. Method of solution

Two point boundary value problem represented by Eqs. (6)–(8) is tackled by implicit finite difference scheme in combination with the quasilinearization technique. Quasilinearization technique can be viewed as a generalization of the Newton–Raphson approximation technique in functional space. An iterative sequence of linear equations are carefully constructed to approximate the nonlinear Eqs. (6) and (7) for achieving quadratic convergence and monotonicity.The unique feature of quasilinear implicit finite difference scheme as quadratic convergence and monotonicity has been found superior than the built-in iteration of upwind technique or finite amplitude technique.The efficiency and accuracy of the method have been illustrated through it's applications to many boundary value problems in the book by Bellman and Kalaba [16].

Applying the quasilinearization technique [12,16,17], we replace the nonlinear coupled partial differential equations (6) and (7) to the following sequence of linear partial differential equations:

$$X_1^i F_{\eta\eta}^{i+1} + X_2^i F_\eta^{i+1} + X_3^i F^{i+1} + X_4^i F_\xi^{i+1} = X_5^i \quad (11)$$

$$Y_1^i G_{\eta\eta}^{i+1} + Y_2^i G_\eta^{i+1} + Y_3^i G_\xi^{i+1} + Y_4^i F_\eta^{i+1} + Y_5^i F^{i+1} = Y_6^i \quad (12)$$

The coefficient functions with iterative index $i$ are known and the functions with iterative index $i+1$ are to be determined. The boundary conditions become

$$\begin{aligned} &F^{i+1} = 0, \quad G^{i+1} = 1 \quad \text{at } \eta = 0 \\ &F^{i+1} = 2, \quad G^{i+1} = 0 \quad \text{at } \eta = \eta_\infty \end{aligned} \quad (13)$$

where $\eta_\infty$ is the edge of the boundary layer. The coefficients in Eqs. (11) and (12) are given by

$$\begin{aligned} &X_1^i = (1+\xi\eta), && Y_1^i = \frac{(1+\xi\eta)}{Pr} \\ &X_2^i = \xi + f + \xi f_\xi, && Y_2^i = \frac{\xi}{Pr} + f + \xi f_\xi \\ &X_3^i = -\xi F_\xi, && Y_3^i = -\xi F \\ &X_4^i = -\xi F, && Y_4^i = 2Ec(1+\xi\eta)F_\eta \\ &X_5^i = -\xi F F_\xi, && Y_5^i = -\xi G_\xi \\ & && Y_6^i = Ec(1+\xi\eta)F_\eta^2 - \xi F G_\xi \end{aligned}$$

Now, at each iteration step, the resulting sequence of linear partial differential equations (11) and (12) under the boundary conditions (13) have been solved numerically using an implicit finite difference scheme. The sequence of linear partial differential equations (11) and (12) were expressed in difference form using central difference scheme in $\eta$-direction and backward difference scheme in $\xi$-direction. In each iteration step, the equations were then reduced to a system of linear algebraic equations with a block tri-diagonal structure which is solved by using Varga algorithm [18]. To ensure the convergence of the numerical solution to exact solution, the step sizes $\Delta\eta$ and $\Delta\xi$ have been optimized and the results presented here are independent of the step sizes at least up to the fourth decimal place. A convergence criteria based on the relative difference between the current and previous iteration values of the velocity and temperature gradients at wall are employed. When the difference reaches less than $10^{-4}$, the solution is assumed to have converged and the iterative process is terminated.

## 4. Results and discussion

Computations have been carried out for various values of $Pr(0.7 \leqslant Pr \leqslant 7.0)$, $Ec(-0.3 \leqslant Ec \leqslant 0.3)$ and $A(-0.6 \leqslant A \leqslant 0.4)$. The step sizes in the $\xi$- and $\eta$-directions have been choosen as $\Delta\xi = 0.05$ and $\Delta\eta = 0.01$ throughout the computations. Results displayed in [7–9] show a general trend that there is a sharp change in shear stress distribution at the leading and trailing edges of the slot due to their choice of mass transfer distributions. Further, the shear stress decreases (increases) for injection (suction) as the slot starts and shear stress reaches its minimum (maximum) value before the trailing edge of the slot. It may be remarked that the present study also shows similar trend as mentioned above except for a sharp change at the leading and trailing edges of the slot due to the choice of the present mass transfer distribution with the computational advantage for higher mass transfer parameter. In order to validate our method, we have compared results of skin friction and heat transfer parameters $(F_\eta(0,0), G_\eta(0,0))$ with those of Takhar et al. [6] and Chen and Mucoglu [2]. The results are found in an excellent agreement and for the sake of briefness, possible comparisons have been shown here in Table 1.

The effects of non-uniform slot suction (or injection) parameter ($A > 0$ or $A < 0$) on the skin friction coefficient and heat transfer parameter $(C_f(Re_x)^{1/2}, Nu(Re_x)^{-1/2})$ are presented in Figs. 2 and 3. In case of non-uniform slot suction, the skin friction coefficient and heat transfer parameter $(C_f(Re_x)^{1/2}, Nu(Re_x)^{-1/2})$ increase as the slot starts and attain their maximum values in the middle of the slot and decrease from their maximum values towards the trailing edge of the slot. Figs. 2 and 3 also show that the effect of non-uniform slot injection is just opposite. Hence, non-uniform slot injection helps to reduce skin friction and heat transfer coefficients at a particular streamwise location on the slender cylinder body surface.

Table 1
Comparison of heat transfer results ($NuRe_x^{-1/2}$) with those of Takhar et al. [6]

| $\xi$ | Present results | | Takhar et al. [6] | |
|---|---|---|---|---|
| | $Pr = 0.7$ | $Pr = 1.0$ | $Pr = 0.7$ | $Pr = 1.0$ |
| 0.005 | 0.36012 | 0.45210 | 0.36261 | 0.45613 |
| 0.01 | 0.36902 | 0.46270 | 0.37054 | 0.46427 |
| 0.04 | 0.40320 | 0.49712 | 0.40504 | 0.49895 |
| 0.05 | 0.41310 | 0.50710 | 0.41443 | 0.50817 |
| 0.06 | 0.42290 | 0.51508 | 0.42328 | 0.51667 |

For the values of $\Omega = 0$ and $\lambda = 0.5$ in [6].

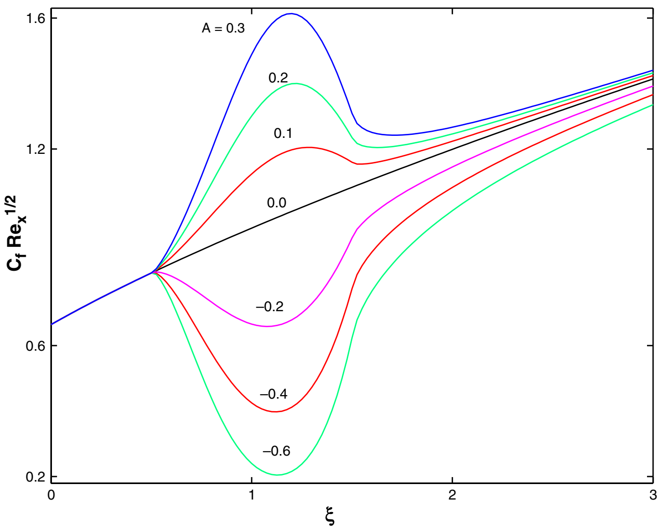


Fig. 2. Effects of non-uniform slot injection ($A < 0$) and suction ($A > 0$) on $C_f(Re_x)^{1/2}$, where $Ec = 0.1$ and $Pr = 0.7$.

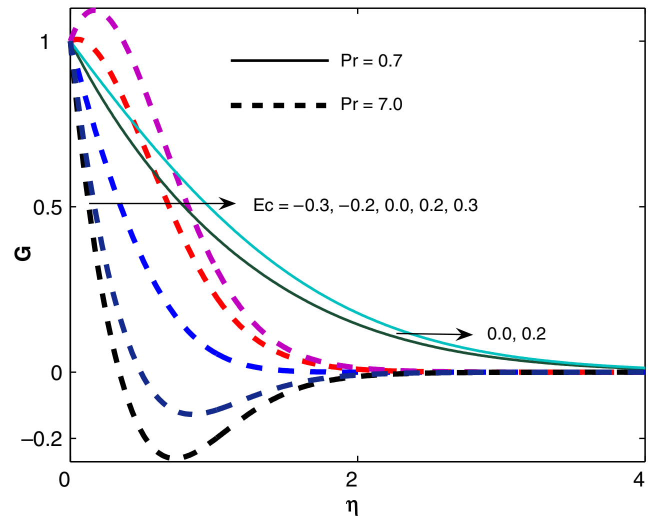


Fig. 4. Effects of $Ec$ and $Pr$ on temperature profile ($G$), where $A = 0$ and $\xi = 1$.

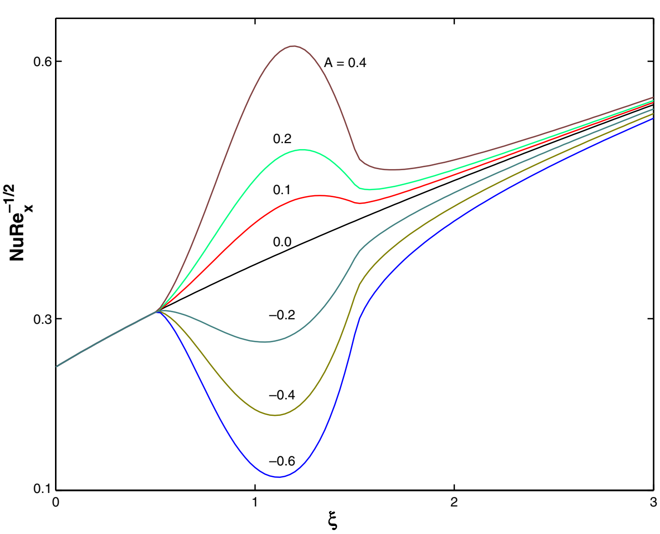


Fig. 3. Effects of non-uniform slot injection ($A < 0$) and suction ($A > 0$) on $Nu(Re_x)^{-1/2}$, where $Ec = 0.1$ and $Pr = 0.7$.

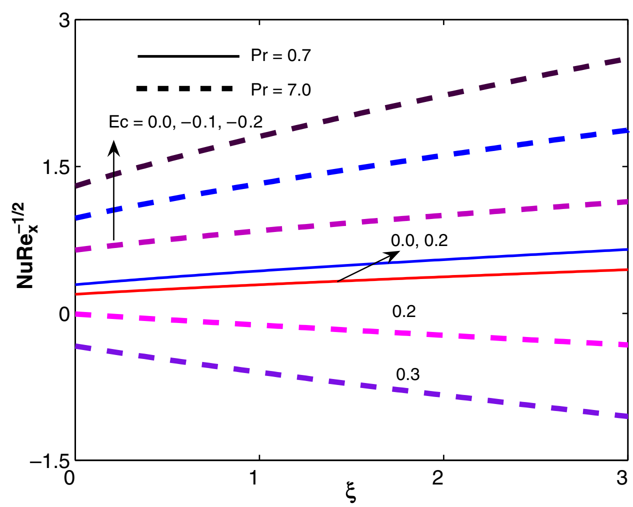


Fig. 5. Effects of $Ec$ and $Pr$ on $Nu(Re_x)^{-1/2}$, where $A = 0$.

Figs. 4 and 5 display the effects of Prandtl number and Eckert number on the temperature profiles ($G$) and heat transfer coefficient $Nu(Re_x)^{-1/2}$, respectively. It is found from Fig. 5 that the surface heat transfer rate increases significantly with $Pr$ as the higher $Pr$ number fluid has a lower thermal conductivity, which results in thinner thermal boundary layer (see Fig. 4) and hence a higher heat transfer rate at the wall. Fig. 5 shows that due to the increase of viscous dissipation parameter $Ec$, heat transfer parameter $Nu(Re_x)^{-1/2}$ decreases. In the case of high Prandtl number fluid ($Pr = 7.0$), for $Ec > 0$, the temperature profile ($G$) exhibits overshoot within the boundary layer before reaching zero value at the edge of the boundary layer (see Fig. 4). This implies that the temperature of the fluid near the wall is greater than that at the wall. Due to viscous dissipation, the fluid near the wall heats up and its temperature become more than the wall, although the wall is maintained at constant higher temperature. Thus, the cooler free stream is unable to cool the hot wall due to the "heat cushion" provided by frictional heating, therefore the wall instead of being cooled will get heated. Moreover, for $Ec > 0$ and $Pr = 7.0$, Fig. 5 displays that $Nu(Re_x)^{-1/2}$ becomes negative indicating the reversal of the direction of heat transfer, i.e., from the edge of the wall to fluid, instead of fluid to wall (which are also observed by the corresponding overshoot in temperature profiles). Similarly, for $Ec < 0$ (i.e., cooled cylinder $T_w < T_\infty$), the temperature profile ($G$) exhibits undershoot within the boundary layer before reaching zero value at the edge of the boundary layer (see Fig. 4). This means that the temperature of the fluid within the boundary layer attains greater value than free stream temperature

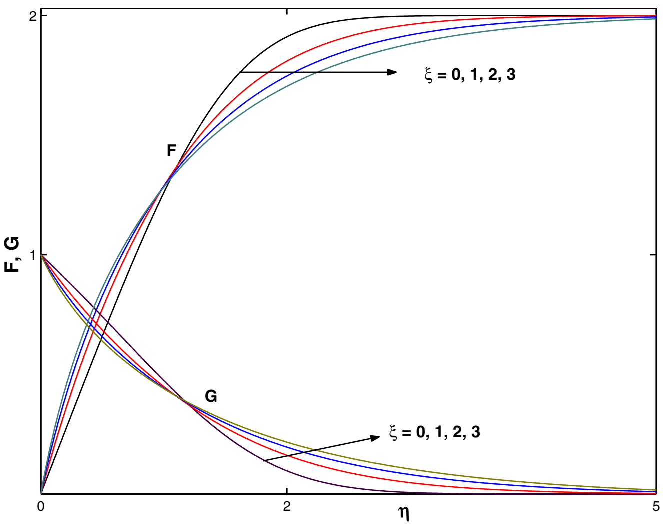


Fig. 6. Effect of transverse curvature parameter ($\xi$) on $F$ and $G$, where $Ec = 0.1$, $A = 0$ and $Pr = 0.7$.

which is again due to the viscous dissipation heating as explained above in details. In fact, at $\xi = 1.0$, the heat transfer coefficient $(Nu(Re_x)^{-1/2})$ reduces from 0.8 to $-0.55$ when $Ec$ varies from 0 to 0.3. However, in the absence of viscous dissipation ($Ec = 0$), heat transfer takes place in the usual way (i.e., from wall to the fluid).

The effects of transverse curvature parameter $\xi$ (or the axial distance) on velocity and temperature profiles $(F, G)$ are displayed in Fig. 6. Due to the increase in surface curvature parameter $\xi$, the steepness in velocity and temperature profiles $(F, G)$ near the wall increases. The physical reason is that the increase in $\xi$ acts as a favourable pressure gradient, which enhances the steepness in velocity and temperature profiles near the wall resulting in higher skin friction and heat transfer rate at wall.

## 5. Conclusions

Non-similar solution of a steady incompressible boundary layer flow over a horizontal slender cylinder with non-uniform slot injection (suction) has been obtained from the origin of streamwise coordinate. Skin friction and heat transfer rate are found to alter significantly due to non-uniform slot injection (or suction). Transverse curvature effect produces higher skin friction and heat transfer rate at the wall. The heat transfer rate is found to depend strongly on viscous dissipation, but it has comparatively very less effect on the skin friction coefficient.

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